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CMSC207

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**Proof Questions ( Homework 10 assignments for chapter 10)**

**207 Chapter 10 “Graphs and Trees” Proof Questions**

**#1.**

Prove that having *n* vertices, where *n* is a positive integer, is an invariant for **graph isomorphism**.

Since an isomorphism of graphs G and H is a bijection of their respective vertex sets, V(G) and V(H), all that is taken into account are the vertices. Thus if G has n vertices and G is isomorphic to H, then H also has n vertices, which means having n vertices is an invariant for graph isomorphism

**#2.**

Prove that the sum of the degrees of the vertices of any finite graph is even.

If you have a graph consisting of only vertices, then the sum of the degrees is 0 which is even. If you add one edge connecting two vertices, the sum of the degrees is 2, which is even. If you add another edge to a vertex that connects to itself, you must count that vertex as having degree 2, which would increase the sum by 2, keeping it even. The sum is always incremented by 2, thus the sum of the degrees of a finite graph is even

**#3.**

Show that every simple finite graph has two vertices of the same degree.

By the pigeonhole principle, if you have n vertices, each of those vertices can be connected to n-1 other vertices, so at least 2 vertices will have the same degree.